

# Control of buckling in large micromembranes using engineered support structures

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## Abstract

In this paper we describe a general method to avoid stress-induced buckling of thin and large freestanding membranes. We show that using properly designed supports, in the form of microbeams, we can reduce the out-of-plane deflection of the membrane while maintaining its stiffness. As a proof of principle, we used a silicon-on-insulator (SOI) platform to fabricate 30  $\mu\text{m}$  wide, 220 nm thick, free-standing Si membranes, supported by four 15  $\mu\text{m}$  long and 3  $\mu\text{m}$  wide microbeams. Using our approach, we are able to achieve an out-of-plane deformation of the membrane smaller than 50 nm in spite of 39 MPa of compressive internal stress. Our method is general, and can be applied to different material systems with compressive or tensile internal stress.

(Some figures may appear in colour only in the online journal)

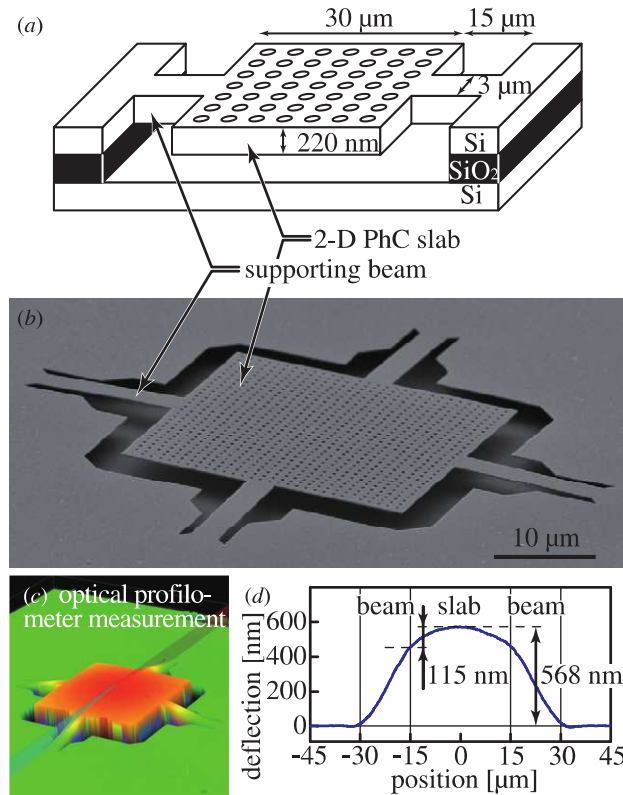
## 1. Introduction

Freestanding micro membrane/beam structures are commonly used in microelectromechanical systems (MEMS) to realize variable capacitors [1], RF switches [1, 2], pressure sensors [3], mechanical memories [4] and movable mirrors [5]. In such freestanding structures, deformation by residual internal stress is a well known and common issue [6–8]. The stress, caused by the thermal expansion coefficient mismatch between different materials, is often built-in during the process of thin film deposition and is therefore difficult to avoid. This is a problematic since large compressive or tensile stresses can lead to buckling or breaking of the membrane. The degree of buckling depends on the length/thickness ratio of a membrane structure: if the ratio is 100 or larger, buckling can be significant even for low values of compressive stress (on the order of several tens of MPa). To reduce the effect of the

internal stress, several stress-reduction techniques have been proposed, including optimization of the deposition conditions [9], postdeposition annealing [10], stress-compensation by a deposition of another film [5] and ion implantation [10, 11]. All of these stress compensation techniques, however, suffer two major drawbacks. First, they are material specific. Second, they only work in a narrow temperature range due to thermal expansion coefficients. In this paper, we describe a general method to control the out-of-plane strain and stiffness of suspended membranes by using properly designed supporting beam structures. We use this method to realize large, free-standing Si membranes perforated with a lattice of holes to define the photonic crystal (PhC) structure shown in figure 1.

Recently, there has been increasing interest in realizing PhC membranes that support guided resonances for applications in the field of optomechanics [12–18]. These membranes are typically fabricated in semiconductor materials exhibiting large dielectric permittivities, e.g. Si or InGaAs.

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**Figure 1.** (a) Schematic image of a target structure with the typical lengths indicated. The structure consists of a thin and large 2D PhC slab supported by four beams. (b) SEM image of the structure. (c), (d) Built-in internal stress causes buckling of the photonic crystal membrane.

To ensure strong optomechanical interaction, thin ( $\sim 200$  nm) and large ( $\sim$ several tens of  $\mu\text{m}$ ) membrane structures, with length/thickness ratios  $\sim 100$ , are desired. The small thickness allows for greater flexibility of the membrane (to enable strong response of the structure to the weak forces), and ensure single-mode (optical) operation in the perpendicular direction, while the large area is needed to ensure good confinement of guided resonances supported by the PhC membrane. Furthermore, structures have been realized with membrane–substrate separations as small as a few hundred nanometers [13, 15–17] in order to enhance the optomechanical coupling. For all of these reasons, and to avoid the failure of the structure, it is essential to reduce buckling of the membrane due to built-in internal stress.

## 2. Design

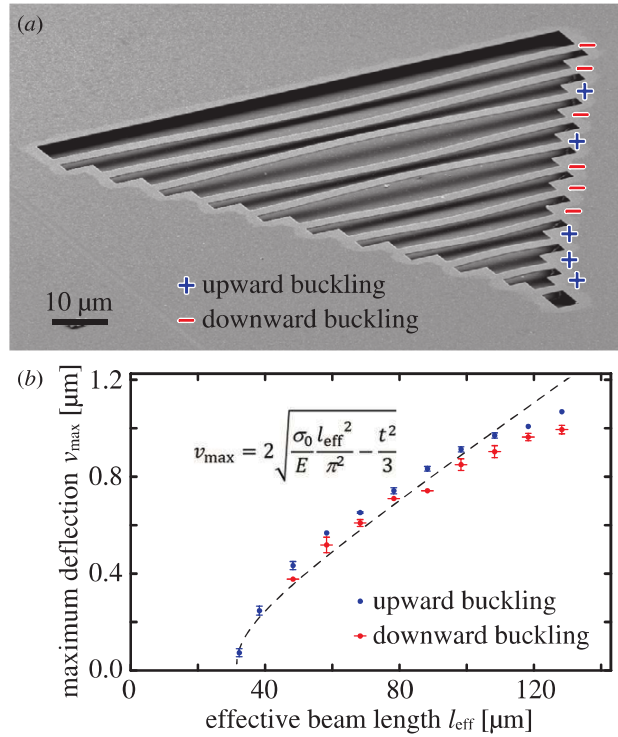
A 2D PhC slab structure that supports guided resonances is shown in figure 1(a). It consists of a 220 nm thick free-standing Si square membrane of width 30  $\mu\text{m}$ , supported by four microbeams of length 15  $\mu\text{m}$ . The membrane is perforated with a PhC lattice of holes with periodicity  $a = 1$   $\mu\text{m}$  and hole radius  $r = 200$  nm. The structure was realized in a commercial SOI wafer (SOITEC Inc.) with 220 nm thick Si device layer and 2  $\mu\text{m}$  thick buried oxide (BOX) layer. The structure was fabricated using electron beam lithography

followed by inductive coupled plasma reactive ion etching (ICP RIE) and HF vapor etching (figure 1(b)). As shown in the optical profilometer measurement in figures 1(c) and (d), the 2D PhC slab structure with simple straight supporting beams is buckled upward by its own internal stress.

In the case of a simple beam structure with length  $l$ , width  $w$  and thickness  $t$ , a critical buckling stress  $\sigma_c$  is proportional to  $(t/l)^2$ . In the case of doubly clamped beams, the critical buckling stress is given by  $\sigma_c = \frac{\pi^2}{3} E \left(\frac{t}{l}\right)^2$ , where  $E$  is the Young's modulus of the structure [19]. (Note: We describe a compressive stress as positive and a tensile stress as negative in this paper.) If the structure has a compressive internal stress  $\sigma_0$  which is larger than the critical buckling stress ( $\sigma_0 \geq \sigma_c$ ), then we can define a critical buckling length  $l_c = \pi t \sqrt{\frac{E}{3\sigma_0}}$ , so that all the beams with length  $l > l_c$  will be buckled. A critical buckling length  $l_c$  of our SOI wafer for a  $t = 200$  nm thick device layer can be estimated to be in the range of  $l_c = 13\text{--}41$   $\mu\text{m}$ , given the internal stress in a typical commercial SOI wafer of  $\sigma_0 = 10\text{--}100$  MPa and the Young's modulus  $E = 169$  GPa for Si in the  $\langle 110 \rangle$  direction [20]. Therefore, buckling can be large even in a structure that is only a few tens of  $\mu\text{m}$  in length (figure 2(a)). We note that the critical buckling length increases to  $l_c = 196\text{--}620$   $\mu\text{m}$  if the thickness of the Si device layer is increased to  $t = 3$   $\mu\text{m}$ . To evaluate the built-in stress in the wafers used in this experiment, we fabricated arrays of doubly clamped beams with varying length and measured their out-of-plane deflection using a 3D optical profilometer (Olympus LEXT OLS4000, display resolution: 1 nm). The results, shown in figure 2(b), are in good agreement with the theoretical model discussed above. Using these values, we estimated the compressive internal stress of the Si device layer of the SOI to be 39 MPa, resulting in the critical buckling length  $l_c = 26.3$   $\mu\text{m}$ . This explains the significant buckling of the PhC membrane (figure 1), whose total length is larger than  $l_c$ .

In order to overcome this buckling issue, it is necessary to redesign the supporting beams. Fundamentally, buckling appears when a compressive internal stress acts as an axial load to a supporting beam (figure 3(a)). Therefore, our design methods are classified into three categories based on the type of the deflection induced by the compressive stress. The ‘type 1’ method (figure 3(b)), leads to in-plane buckling instead of out-of-plane buckling due to axial load by means of the compressive internal stress. The ‘type 2’ method (figure 3(c)), leads to in-plane deformation without buckling due to axial load by taking advantage of beams with an initial deflection (i.e. curved beams). The type 2 beams can be deformed below the in-plane critical buckling stress, whereas type 1 beams are deformed only by stresses above the in-plane critical buckling stress. The ‘type 3’ method (figure 3(d)) uses a lateral beam to transform the compressive stress into a lateral load. All approaches can be described as methods of releasing the internal stress by in-plane deformation.

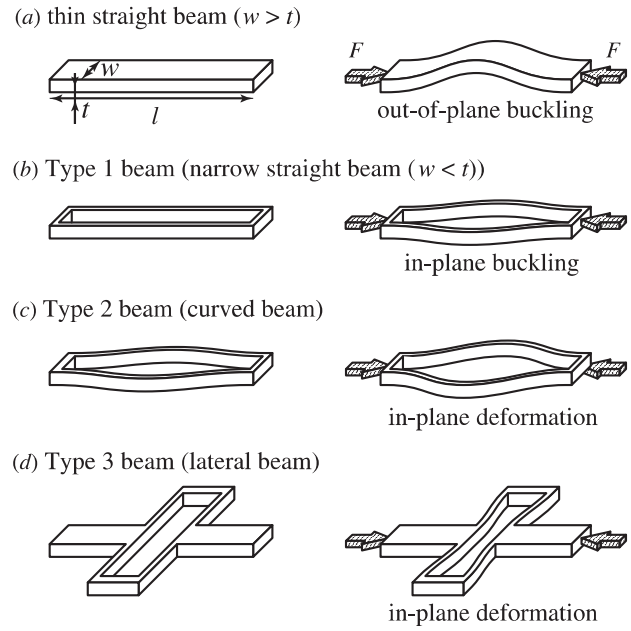
For the type 1 method to work, it is necessary for the in-plane critical buckling stress  $\frac{\pi^2}{3} E \left(\frac{w}{l}\right)^2$  to be smaller than the out-of-plane critical buckling stress  $\frac{\pi^2}{3} E \left(\frac{t}{l}\right)^2$ . Therefore, an extremely narrow and long beam is required which may be challenging to implement. In addition, type 1 is only effective



**Figure 2.** Characterization of stress-induced deformation in a commercial SOI wafer with  $t = 220$  nm thick device layer. (a) The doubly clamped beams with different lengths are buckled upward or downward due to the internal stress. (b) Deflection of each beam as a function of its length. Eleven beams with lengths  $l = 4, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$  μm have been evaluated. The horizontal axis is the effective beam length  $l_{\text{eff}}$  which is the sum of the designed beam length  $l$  and a constant undercut length. We estimate the undercut length to be 23 μm. Blue and red dots are deflections of upward and downward buckling, respectively. Error bars refer to the standard deviations of the deflection measurement. The slight difference between the amount of upward and downward buckling can be attributed to the stress gradient and bending moment at the ends. The dashed line is a fitting curve of a theoretical deflection  $v_{\max} = 2 \sqrt{\frac{\sigma_0 l_{\text{eff}}^2}{E \pi^2} - \frac{l^2}{3}}$  [5] with  $\sigma_0$  of 39 MPa.

in the case of compressive stress, while types 2 and 3 can be used for both compressive and tensile stress.

Types 2 and 3 are based a similar idea: the stiffness of a simple straight beam against an axial load (i.e. in-plane stiffness) is  $E \frac{wt}{l}$ . Therefore, if part of the beam has in-plane stiffness smaller than  $E \frac{wt}{l}$ , it can relieve the internal stress by elastic deformation. However, we do not want to reduce the out-of-plane stiffness (i.e. out-of-plane spring constant), because the supporting beams have to play two roles: (1) reducing the out-of-plane deformation and (2) supporting the membrane structure. Therefore we use a multiple-narrow-beam structure, shown in figure 4. Figure 4 shows the in-plane and out-of-plane stiffness of a simple straight beam and type 3 beams. Using a multiple-narrow-beam structure it is possible to reduce the in-plane stiffness while maintaining the out-of-plane stiffness of a simple straight beam. That is, if we change from a simple lateral beam to  $n$  beams each with a fraction  $1/n$  of the original width, we obtain  $1/n^2$  times smaller stiffness in



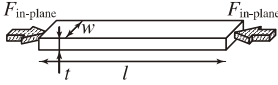
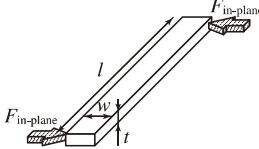
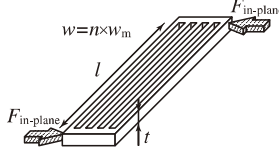
**Figure 3.** Different beam structures that can be used to overcome out-of-plane buckling due to an axial load. All approaches are based on inducing in-plane deformation to overcome the out-of-plane deformation due to an axial load.

the in-plane direction while maintaining the same out-of-plane stiffness (shown in figure 4). The same analysis applies to type 2 beams, and is shown in figure 5.

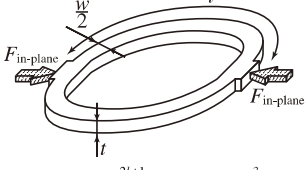
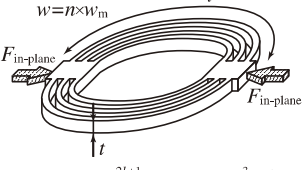
### 3. Fabrication and evaluation

To verify our designs, we fabricated the structures discussed in the previous section, using the same SOI wafer as in figure 2. The SEM images and the beam deflections measured using a 3D optical profilometer are shown in figure 6. The beams have length 100 μm, width 3 μm, and thickness 220 nm. Figure 6(c) shows a simple straight beam with 1212 nm of out-of-plane deflection due to buckling. Figure 6(e) shows a multiple-narrow-beam type 2 structure that consists of 8 narrow curved beams with 200 nm widths. Figure 6(f) shows a multiple-narrow-beam type 3 structure that consists of 15 narrow lateral beams with 200 nm widths. The maximum out-of-plane deflections of the multiple-narrow-beam types 2 and 3 structures are 280 nm and 114 nm, respectively. As can be seen, the out-of-plane deflections are highly reduced using our proposed methods. Figure 6(d) is a beam with multiple-narrow straight beams without initial deflection, so the beam also suffers out-of-plane buckling of 1211 nm. Though we could not measure the in-plane deformation in figure 6(e), we can obviously confirm the effect of initial deflection (i.e. curved beam) by comparing to the out-of-plane deflection of figure 6(d).

Figures 1(b) and 7(a), (b) show the photonic crystal membrane structures with simple, types 2 and 3 supporting beams, respectively. Using the profilometer, we measured the maximum upward deflections of types 2 and 3 to be 49 and 33 nm, respectively. This is more than an order of magnitude

	simple straight beam	Type 3 beam (lateral beam)	multiple-narrow-beam Type 3 structure (multiple-narrow lateral beams)
			
in-plane stiffness	$E \frac{wt}{l}$	$a \times E \frac{w^3 t}{l^3}$	$a \times E \frac{n w_m^3 t}{l^3} = a \times E \frac{w^3 t}{l^3} \frac{1}{n^2}$
out-of-plane stiffness	$b \times E \frac{wt^3}{l^3}$	$b \times E \frac{wt^3}{l^3}$	$b \times E \frac{n w_m t^3}{l^3} = b \times E \frac{wt^3}{l^3}$

**Figure 4.** In-plane and out-of-plane stiffness of a simple straight beam and type 3 beams. Stiffness is defined by a load/deformation ratio (i.e.  $F/v_{\max}$ ). In the case of a simple straight beam, the in-plane stiffness (for the axial load) is  $E \frac{wt}{l}$  and the out-of-plane stiffness is proportional to  $E \frac{wt^3}{l^3}$ . The coefficients  $a$  and  $b$  are determined by load and end conditions. For example, in the case of a doubly clamped beam under point loading at the center,  $a = b = 16$ . In the case of a guided-clamped beam under point loading at the free end,  $a = b = 1$  [21].

	Type 2 beam (curved beam)	multiple-narrow-beam Type 2 structure (multiple-narrow curved beams)
		
in-plane stiffness	$\sum_{k=1}^{\infty} a_k \times E \frac{w^{2k+1} t}{l^{2k+1}} \approx a_1 \times E \frac{w^3 t}{l^3}$	$\sum_{k=1}^{\infty} a_k \times E \frac{n w_m^{2k+1} t}{l^{2k+1}} \approx a_1 \times E \frac{w^3 t}{l^3} \frac{1}{n^2}$
out-of-plane stiffness	$b \times E \frac{wt^3}{l^3}$	$b \times E \frac{n w_m t^3}{l^3} = b \times E \frac{wt^3}{l^3}$

**Figure 5.** In-plane and out-of-plane stiffness of type 2 beams. We can use the same analysis as with type 3 under the assumption that the length/width ratio is large.

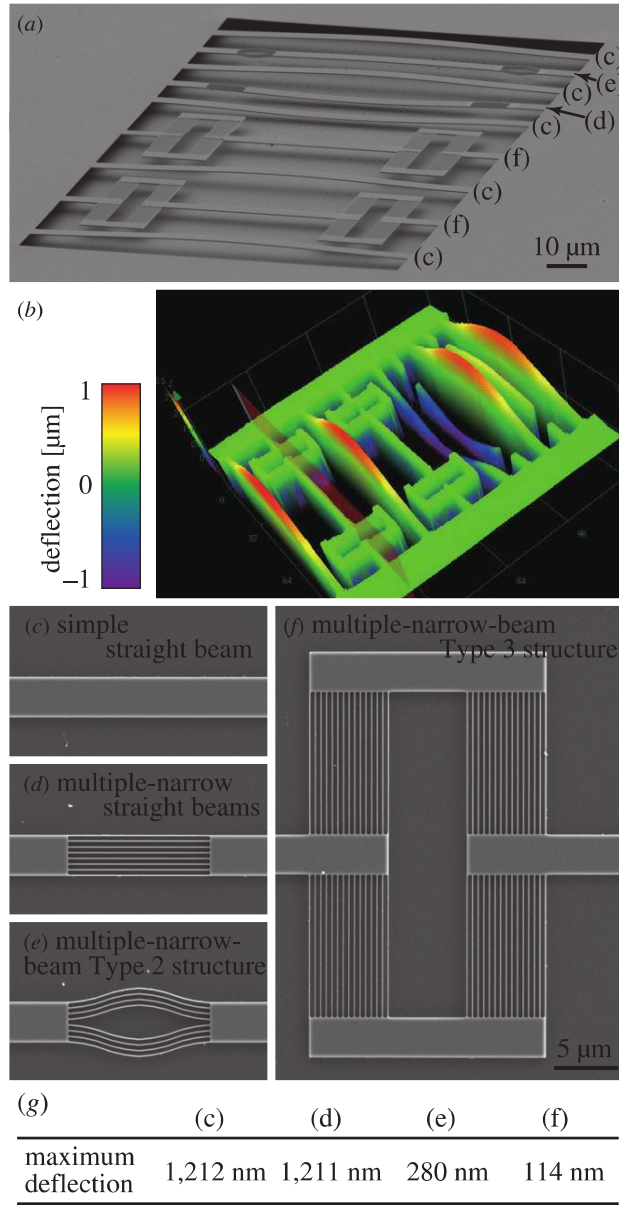
smaller than a maximum deflection (568 nm) measured for the membrane with simple supporting beams (figure 1(b)). Finally, we note that since the supporting beams relieve the internal stress, the membrane itself does not bend (up to the resolution of our apparatus) under its internal stress, and is very flat. This is of great importance for many photonic applications, including movable mirrors and variable capacitors.

#### 4. Discussion

Approaches to reduce the out-of-plane buckling/deflection by designing proper supports, using methods that fall in to type 2 [22] or type 3 [23, 24] categories, have previously been studied. The novel aspect of our work is the application of multiple-narrow-beam structures that offer very important advantages over other methods: the in-plane and out-of-plane stiffness can be designed almost independently. That is, by increasing  $n$ , we can reduce the in-plane stiffness of the beam without changing the out-of-plane stiffness, and thus not change the spring constant of the membrane. This is important to designs of related devices such as variable capacitors, RF switches and movable mirrors.

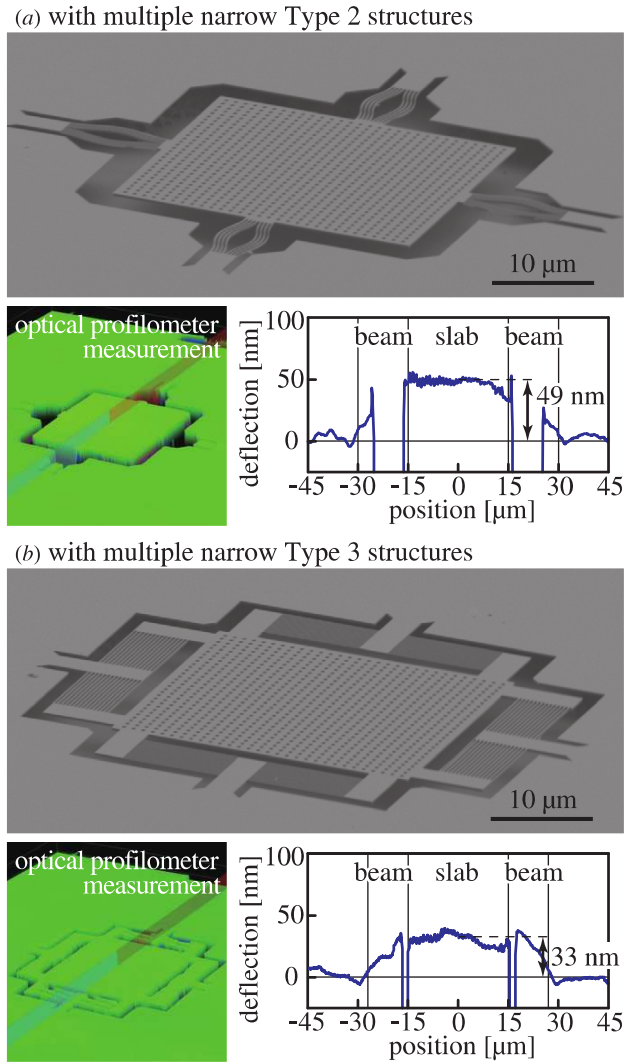
In the case of a simple beam, the internal stress before ( $\sigma_0$ ) and after ( $\sigma_f$ ) releasing are the same, up to the critical buckling stress  $\sigma_c$  in compression or the breaking stress  $\sigma_b$  in tension (figure 8). Therefore the simple beam can be used as long as the internal stress  $\sigma_0$  satisfies  $\sigma_b < \sigma_0 < \sigma_c$ . In the case of type 2 or type 3 beams, with smaller in-plane stiffness,  $\sigma_0$  and  $\sigma_f$  are different due to the in-plane elastic deformation of the beam. Therefore, we define two new parameters, the highest allowable compressive stress  $\sigma_{AC}$  and the highest allowable tensile stress  $\sigma_{AT}$ , in the following way: if the initial stress  $\sigma_0$  is  $\sigma_{AC}$  or  $\sigma_{AT}$ , the final stress  $\sigma_f$  would be  $\sigma_c$  or  $\sigma_b$ , respectively (see figure 8). As a result, types 2 and 3 beams can be used as long as the initial internal stress  $\sigma_0$  satisfies  $\sigma_{AT} < \sigma_0 < \sigma_{AC}$ . In the case of the multiple-narrow-beam structure,  $\sigma_{AC}$  and  $\sigma_{AT}$  are proportional to  $n^2$ . Therefore it is always possible to choose  $n$  satisfying this condition, which makes our approach general and useful for a range of geometries and films deposited using different techniques. However, in practice the upper limit of  $n$  is set by the minimum resolution of the fabrication process; the larger the  $n$ , the smaller the width of each beam in the multibeam section. If the characteristic beam width is  $w_0$  and the minimum available width on the fabrication is  $w_{\min}$ ,  $n$  should be smaller





**Figure 6.** Doubly clamped beams with different support beam structures. All beams are  $100\ \mu\text{m}$  long and  $3\ \mu\text{m}$  wide. (a) SEM image of the beams. Note that the labels in this figure correspond to the SEM images below. (b) Measurement of the deflections obtained using a 3D optical profilometer. (c)–(f) Enlarged images of each beam structure before releasing. (g) Table showing the maximum deflection of the beams.

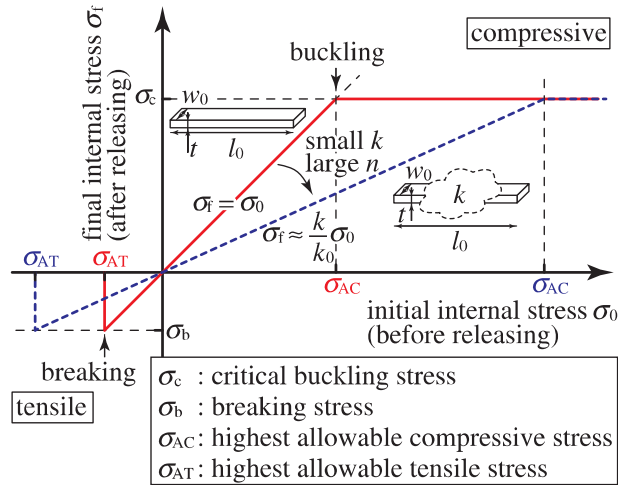
than  $w_0/w_{\min}$ . For example our characteristic beam width was  $3\ \mu\text{m}$  and the minimum resolution is approximately  $50\ \text{nm}$ , because of our use of electron beam lithography, the upper limit of  $n$  is 60. As mentioned above, the allowable stresses  $\sigma_{AC}$  and  $\sigma_{AT}$  are proportional to  $n^2$ , so our method in this geometry can be used to deal with a wide range of internal stress. This is another unique feature of our method that has not been previously investigated. Our approach is material-platform nonspecific which further adds to its versatility. In addition, because our method can release internal stress of any



**Figure 7.** 2D PhC slab structures with supporting beams. The period and hole radius of the PhC slab is  $1\ \mu\text{m}$  and  $200\ \text{nm}$ , respectively. The dimension of the slab is  $30\ \mu\text{m}$  by  $30\ \mu\text{m}$ , and characteristic length and width of the supporting beams are  $15\ \mu\text{m}$  and  $3\ \mu\text{m}$ , respectively. (a) and (b) are structures with multiple-narrow-beam type 2 and 3 structures, respectively. Structure with simple beams, fabricated on the same wafer, is shown in figure 1(b). The ‘noise’ in the measurement at the position of the slab is due to the photonic crystal holes with sizes below the resolution limit of the profilometer.

magnitude as long as the stress is smaller than the allowable internal stresses, we do not have to know the precise internal stress of the membrane. In other words, if we choose a large enough value  $n$ , our beams make proper in-plane deformations according to the magnitude of internal stress automatically. This is very effective, especially for mitigating of temperature-induced stress whose value can greatly vary with temperature: our approach’s ‘self-adaptability’ to internal stress makes it effective for a wide range of temperatures.

Though our method highly reduces the out-of-plan deflection, the structures still have finite deflection  $<50\ \text{nm}$ . We believe that this is due to the stress gradient of the



**Figure 8.** Illustration of the proposed stress-control approach based on engineered support structures. The released membrane structure is flat (no buckling) when the final internal stress after releasing ( $\sigma_f$ ) is between  $\sigma_c$  and  $\sigma_b$ , where  $\sigma_c$  and  $\sigma_b$  are the critical buckling stress and the breaking stress, respectively. Corresponding to an initial internal stress before releasing  $\sigma_0$ ,  $\sigma_0$  should have a value between  $\sigma_{AC}$  and  $\sigma_{AT}$ , where  $\sigma_{AC}$  and  $\sigma_{AT}$  are the highest allowable compressive and tensile stress, respectively. The red and blue lines show the cases of a simple straight beam and a beam with a smaller stiffness region, respectively. In the case of a simple straight beam,  $\sigma_{AC}$  and  $\sigma_{AT}$  are the same as  $\sigma_c$  and  $\sigma_b$ , respectively (red line). But the beam has smaller stiffness beam with  $k$  in the in-plane stiffness,  $\sigma_{AC}$  and  $\sigma_{AT}$  is enlarged approximately as  $\frac{k_0}{k} \sigma_c$  and  $\frac{k_0}{k} \sigma_b$ , where  $k_0 = E \frac{w_0^3}{l_0^3}$ ,  $l_0$  is the beam length from end to end, and  $w_0$  is the beam width at the end (blue dashed line). If the multiple-narrow-beam type 2 or type 3 structure is used, the in-plane stiffness  $k$  is described by the formula  $k = \frac{a}{n^2} E \frac{w^3}{l^3}$  (as shown in figures 4 and 5), where  $n$  is the number of nanobeams used in the structure.

membrane and/or a bending moment at its ends, which can also cause asymmetric upward and downward deflections. Indeed, it can be seen in figure 2(b) that the upward and downward deflections are slightly different ( $\sim 80$  nm). The stress gradient comes from the wafer fabrication process and the bending moment at the ends comes from the residue of the sacrificial layer under the membrane. Since the 2D PhC slab region is very flat (in figures 7(a) and (b)) and most of the deflection occurs at the supporting beam region, we can conclude that the residual deflection is not due to the stress gradient, but rather due to the bending moment at the beam ends. Though our method cannot deal with a stress gradient or a bending moment at the ends, these effects cause little deflection in our demonstration.

## 5. Conclusions

We proposed a method to reduce the out-of-plane deflection of a membrane structure due to its internal stress, while maintaining the out-of-plane stiffness of the structure. Using the multiple-narrow-beam type 2 and 3 structures as supporting beams, it is possible to precisely engineer the residual stress simply by changing the number of beams ( $n$ ). Applying our methods to a commercial SOI wafer with 39 MPa of

compressive internal stress, we were able to reduce the out-of-plane deflection of the freestanding membrane by more than an order of magnitude, from 568 nm to below 50 nm. In the cases of type 2 and 3 structures, the maximum deflections were 49 and 33 nm, respectively. The characteristic length and thickness of the membrane structure is 60  $\mu$ m and 220 nm, respectively, so that the length/thickness ratio of our membrane structure is 273 and the maximum deflection/length ratio is less than 1/1000. These values mean the freestanding thin and large membrane structure has little out-of-plane bending. In addition, our method can deal with a wide range of stress values and therefore is suitable for the compensation of not only the compressive but also the tensile internal stress. Furthermore, it can operate in a wide temperature range, since it can handle temperature-induced stress.

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