

# Dynamics of actively mode-locked Quantum Cascade Lasers

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**Abstract:** The impact of upper state lifetime and spatial hole burning on pulse shape and stability in actively mode locked QCLs is investigated by numerical simulations. It is shown that an extended upper state lifetime is necessary to achieve stable isolated pulse formation per roundtrip. Spatial hole burning helps to reduce the pulse duration by supporting broadband multimode lasing, but introduces pulse instabilities which eventually lead to strongly structured pulse shapes that further degrade with increased pumping. At high pumping levels gain saturation and recovery between pulses leads to suppression of mode locking. In the absence of spatial hole burning the laser approaches single-mode lasing, while in the presence of spatial hole burning the mode locking becomes unstable and the laser dynamics does not reach a steady state anymore.

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**OCIS codes:** (140.4050) Mode-locked lasers; (140.5965) Semiconductor lasers, quantum cascade

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## 1. Introduction

Short pulse generation from Quantum Cascade Lasers (QCLs) emitting in the mid-infrared region (3.5–20  $\mu\text{m}$ ) could serve many applications ranging from time-resolved spectroscopy [1,2], and nonlinear frequency conversion [3–5] to high-speed free space communication [6] and frequency metrology [7]. QCLs [8] were first demonstrated in 1994 and they have become the most prominent and compact coherent light source in the mid-infrared. While conventional semiconductor lasers are bipolar devices and have an upper state lifetime ranging from hundreds of ps up to ns, QCLs are unipolar devices with an upper state lifetime that is in the few ps range. In QCLs, the emission wavelength, the gain spectrum and the carrier transport characteristics can be engineered over a wide range of values.

The gain bandwidth of QCLs is large enough to potentially generate sub-picosecond mid-infrared laser pulses due to the flexibility offered by band structure engineering. The most common technique for ultrashort pulse generation is mode locking, in which the longitudinal modes of the cavity are phaselocked either by an internal mechanism (passive mode locking) or by an external (active mode locking). However, short pulse generation from QCLs by mode locking is difficult due to the fast gain recovery time [9,10]. In intersubband transitions (i.e. transitions between quantized conduction band states in semiconductor quantum wells), the carrier relaxation time is very fast due to optical phonon scattering. As a result, the upper state lifetime in QCLs is typically one order of magnitude smaller than the roundtrip time, 40–60 ps, of typical few mm long laser structures [9,10]. Usually, this situation prevents the occurrence of mode locking, i.e. stable pulse formation. The reason is, that the gain fully recovers before the next pulse arrives or even immediately after the pulse if the gain is fast enough, which leads to less amplification of the peak of the pulse than its wings. Due to this process the pulse lengthens in time and the laser approaches continuous wave (cw) lasing. On the other hand, if the gain recovery time is longer than the cavity roundtrip time, the gain saturates with the average power in the laser cavity and does not shape the pulse significantly (Fig. 1). Therefore, most recently, QCLs with long upper state lifetime have been fabricated by implementing a "superdiagonal" gain structure. Upper state lifetimes of 50 ps, similar to the cavity roundtrip time, have been achieved. Using such structures, an actively mode-locked

QCL [11,12] that produced stable and isolated picoseconds pulses, as confirmed by interferometric autocorrelation measurements, was demonstrated. Stable operation with these devices was obtained by current modulation of only a short section of the waveguide, while the whole waveguide was biased slightly above threshold.

In this work, first the impact of upper state lifetime and pumping level on pulse formation is discussed without taking spatial hole burning into account. Then the role of spatial hole burning (SHB) in pulse shaping, destabilization of mode locking and interaction with fast gain recovery and saturation is clarified, beyond what has been discussed previously in [12,13], by extensive numerical simulations. SHB significantly reduces the pulse duration by supporting broadband multimode operation. However, it leads to pulse instabilities and non stationary pulse generation from the laser.

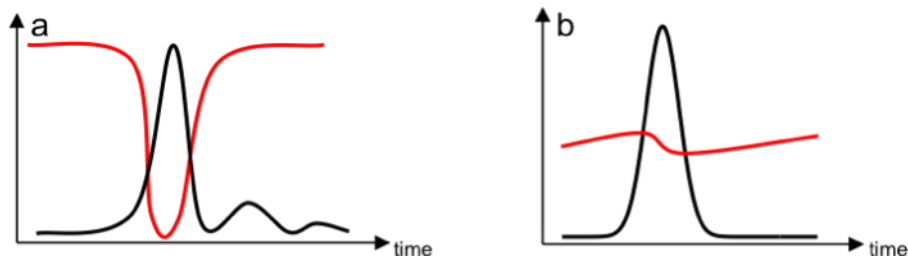


Fig. 1. Intensity (black line) and inversion (red line) for (a) fast gain recovery time and (b) slow gain recovery time

## 2. Prior work on multimode regimes in QCLs

There are several previous works in which multimode regimes in QCLs were observed [6,14–16]. In [14] self mode locking of CW pumped QCLs was claimed. The lasers were emitting broadband optical spectra over a wide range of DC pumping levels. In the photocurrent spectrum a strong, stable and narrow (less than 100 kHz) peak was observed, which indicated long term coherence of phase relationships between the longitudinal modes. However, no autocorrelation data were reported from which one could infer the formation of a stable train of ultrashort pulses separated by the cavity roundtrip time.

In [6] active mode locking in QCLs, by modulating the laser current at the cavity roundtrip frequency, was pursued. Again, evidence of mode locking was deduced from the measured broadband optical spectra, as well as from the power spectra of the photocurrent. When the detuning between the driving frequency and the roundtrip frequency was large, only one mode was lasing. As the detuning was decreased more and more modes were lasing and a narrow beat note at roundtrip cavity frequency appeared. Information about the temporal profile of the generated pulses and proof for true short pulse generation can be obtained with second order autocorrelation measurements. Due to the lack of second order autocorrelators in the mid-infrared at that time, there was no direct evidence that the circulating waveform was consisting of a train of periodic isolated pulses with a stable steady state pulse shape.

Once autocorrelation techniques based on two photon absorption quantum well infrared photodetectors (QWIPs) [17,18] became available, it was discovered that the wideband multimode operation in QCLs is due to phenomena, such as spatial hole burning and the Risken-Nummedal-Graham-Haken (RNGH) instability [13,19]. Recently, it was proposed that self-induced transparency (SIT) together with a fast saturable absorber may be used to passively mode lock QCLs [20].

## 3. Device model and modulation scheme

The actively mode locked QCL, which we will study in the following is schematically depicted in Fig. 2a. A detailed description of a specific device can be found in [12]. The laser

cavity is formed by two semiconductor/air interfaces, which shall be located at  $z = 0$  and  $z = L$ , where  $L$  is the cavity length. The cavity is divided into a short and a long section (Fig. 2b). Both sections are assumed to be equally and continuously pumped (DC pumping) with a current density  $J = p \times J_{th}$ , where  $J_{th}$  denotes the threshold current density and the pump parameter  $p$  shows how many times the laser is pumped above threshold.

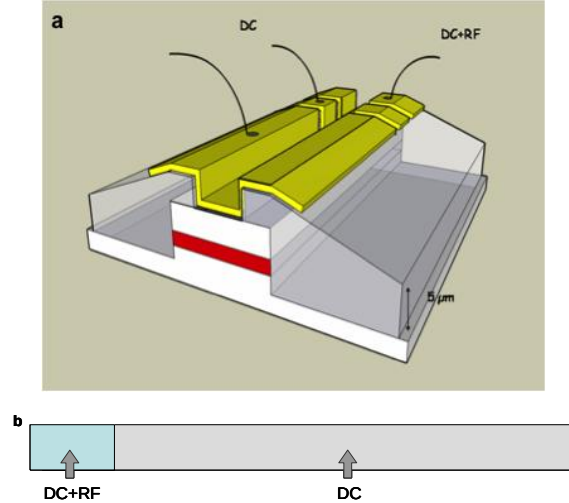


Fig. 2. a) Device used in the experiment, b) Modulation scheme for the actively mode locked QCL. The cavity is in total 2.6 mm long and the short modulator section at the beginning of the cavity is 240  $\mu\text{m}$  long.

Active mode locking is achieved by sinusoidal modulation of the pump current injected into the short, electrically isolated, waveguide section with the roundtrip frequency of the passive cavity. The modulation is supposed to drive a large number of longitudinal modes above threshold by creating modulation sidebands resonant with neighboring modes. The pump current of the small section is then given by  $J = J_{th} \times [p + m \sin(2\pi f_R t)]$ , where  $m$  is the modulation amplitude relative to threshold.

The modulation of the injection current into the short section leads to temporal variation of the roundtrip gain that a pulse can experience, favoring pulsed operation. The common approach to achieve active mode locking is by periodic modulation of the intracavity losses. Here, we modulate the gain. The short section of the cavity acts as a gain modulator, and the long section is pumped to transparency or to slight net gain, helping to compensate the other losses in the laser cavity.

#### 4. Numerical model of QCL

##### 4.1 Three-level versus two-level model

The dynamics of a QCL can be described by a three level model [21] as it is shown in Fig. 3a. It assumes that all QCL stages are identical and that laser action occurs between levels 2 and 1, the upper and lower states of the laser transition. Level 0 corresponds to the superlattice which connects to the next QCL stage, which is again described by the upper laser level 2. The current density  $J$  driven through the device by an external voltage acts as the pump current density from level 0 to level 2.  $T_{21}$  is the lifetime of the upper level,  $T_{10}$  is the lifetime of the lower laser level, that couples to the superlattice. In reality, there is also a superlattice transport time,  $T_{SL}$ , that describes the time it takes for the carriers to travel between different

stages, which we neglect here. The rate equations for the 3-level system shown in Fig. 3a reads

$$\frac{dN_2}{dt} = J - \sigma I_{ph} (N_2 - N_1) - \frac{1}{T_{21}} N_2 \quad (1)$$

$$\frac{dN_1}{dt} = \sigma I_{ph} (N_2 - N_1) + \frac{1}{T_{21}} N_2 - \frac{1}{T_{10}} N_1 \quad (2)$$

$$\frac{dN_0}{dt} = -J + \frac{1}{T_{10}} N_1 \quad (3)$$

where  $N_2$ ,  $N_1$  and  $N_0$  are the population sheet densities of levels 2, 1 and 0 respectively,  $J$  is the pump current density from level 0 to the upper lasing level 2,  $\sigma$  is the cross section for stimulated emission between levels 2 and 1 and  $I_{ph}$  is the photon flux at the transition frequency  $f_{21}$ . If the relaxation rate  $1/T_{10}$  is very large in comparison with the stimulated and spontaneous transition rate from level 2 to 1, which is  $\sigma I_{ph} (1 - N_1/N_2) + 1/T_{21}$ , then level 1 will stay empty at all times, i.e.  $N_1 = 0$ . This is especially the case in the laser here, since it is never pumped far above threshold and the ratio  $T_{21}/T_{10} \approx 100$ . Thus even for sub-picosecond short pulses the stimulated emission rate would never exceed the decay rate  $1/T_{10}$ , which would lead to significant build-up of population in the lower laser level.

Assuming that the lower lasing level stays empty, we can describe the QCL dynamics by an open two-level model (Fig. 3b) for simplicity. The rate equations for the 3-level system shown in Fig. 3b read

$$\frac{dN_2}{dt} = J - \sigma I_{ph} N_2 - \frac{1}{T_{21}} N_2 \quad (4)$$

$$N_1 = 0 \quad (5)$$

The population inversion in the open two-level model is effectively the upper state population. The gain recovery time in the two-level model is then equal to the upper state lifetime  $T_{21}$  and the pump current density fills the upper laser state 2 at a given rate determined by the injected current density. The two-level model does not include the superlattice transport dynamics since it is derived from the three level model (Eq. (1)-(3)) which also does not involve this transport and this may be one reason for some of the remaining discrepancies between the theoretical and experimental results.

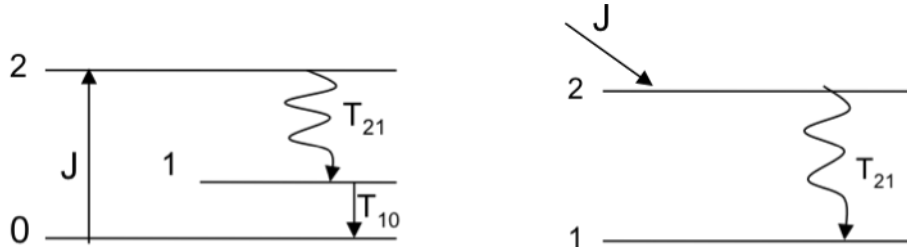


Fig. 3. a) Three-level system which describes QCL dynamics, b) Open two-level model that we use in the simulations

#### 4.2 Maxwell-Bloch Equations

To describe the interaction of the laser field with the gain medium, we expand the two level rate equations to full Maxwell-Bloch Equations [13], that also take the coherent interaction between the field and the medium into account. The dynamics of polarization and inversion of the gain medium is described by the Bloch equations (Eq. (6-7)) and the pulse propagation through the gain medium located in the Fabry Perot cavity is described by the wave equation (Eq. (8)). The electric field propagates in both directions resulting in standing waves.

$$\partial_t \rho_{ab} = i\omega \rho_{ab} + i \frac{dE}{\hbar} \Delta - \frac{\rho_{ab}}{T_2} \quad (6)$$

$$\partial_t \Delta = \lambda - 2i \frac{dE}{\hbar} (\rho_{ab}^* - \rho_{ab}) - \frac{\Delta}{T_1} + D \frac{\partial^2 \Delta}{\partial z^2} \quad (7)$$

$$\partial_z^2 E - \frac{n^2}{c^2} \partial_t^2 E = \frac{Nd}{\epsilon_0 c^2} \partial_t^2 (\rho_{ab} + \rho_{ab}^*) \quad (8)$$

where  $\rho_{ab}$  is the off-diagonal element of the density matrix,  $\Delta \equiv \rho_{bb} - \rho_{aa}$  is the population inversion,  $\omega$  is the resonant frequency of the two-level system,  $d$  is the dipole matrix element of the laser transition,  $D$  is the diffusion coefficient,  $E$  is the electric field,  $N$  is the number of two-level systems per unit volume,  $n$  is the background refractive index,  $T_1$  is the upper state lifetime,  $T_2$  is the dephasing time and  $\lambda$  is the pumping rate, that is directly proportional to the injection current  $J$  in the rate equations. In the model, we modulate the gain in the short section via the pump parameter  $\lambda = \lambda_{th} \times [p + m \sin(2\pi f_R t)]$  and in the long section  $\lambda = \lambda_{th} \times p$ . The last term in Eq. (7) accounts for spatial diffusion of the inversion, i.e. of electrons in the upper laser level along the plane of the layers.

The waves traveling in the two directions are coupled as they share the same gain medium. This gives rise to SHB: the standing wave formed by a cavity mode imprints a grating in the gain medium through gain saturation. As a result, other modes may become more favorable for lasing and multimode operation is triggered.

We make the following ansatz for the electric field, the polarization and the inversion:

$$E(z, t) = \frac{1}{2} [E_+^*(z, t) e^{-i(\omega t - kz)} + E_+(z, t) e^{i(\omega t - kz)}] + \frac{1}{2} [E_-^*(z, t) e^{-i(\omega t + kz)} + E_-(z, t) e^{i(\omega t + kz)}] \quad (9)$$

$$\rho_{ab}(z, t) = \eta_+(z, t) e^{i(\omega t - kz)} + \eta_-(z, t) e^{i(\omega t + kz)} \quad (10)$$

$$\Delta(z, t) = \Delta_0(z, t) + \Delta_2(z, t) e^{2ikz} + \Delta_2^*(z, t) e^{-2ikz} \quad (11)$$

where  $k \equiv \omega n / c$ . The + and – subscripts label the two directions of propagation.  $E$  and  $\eta$  are the slowly varying envelopes in time and space of the electric field and the polarization, respectively. The spatially dependent inversion is written as a sum of three terms, where  $\Delta_0$  is the average inversion and  $\Delta_2$  is the amplitude of the inversion grating.  $\Delta_0$  and  $\Delta_2$  vary slowly in time and space.

We substitute Eq. (9-11) into Eq. (6-8), and perform the slowly varying envelope approximation obtaining the following set of equations [13]:

$$\frac{n}{c} \partial_t E_{\pm} = \mp \partial_z E_{\pm} - i \frac{Nd^2k}{\epsilon_0 n^2} \eta_{\pm} - \ell E_{\pm} \quad (12)$$

$$\partial_t \eta_{\pm} = \frac{id}{2\hbar} (\Delta_0 E_{\pm} + \Delta_2^{\mp} E_{\mp}) - \frac{\eta_{\pm}}{T_2} \quad (13)$$

$$\partial_t \Delta_0 = \lambda + \frac{id}{\hbar} (E_+^* \eta_+ + E_-^* \eta_- - c.c.) - \frac{\Delta_0}{T_1} \quad (14)$$

$$\partial_t \Delta_2^{\pm} = \pm i \frac{d}{\hbar} (E_+^* \eta_- - \eta_+^* E_-) - \left( \frac{1}{T_1} + 4k^2 D \right) \Delta_2^{\pm} \quad (15)$$

For compact notation we introduced:  $\Delta_2^+ \equiv \Delta_2$  and  $\Delta_2^- \equiv \Delta_2^* \cdot (\Delta_2^+)^* \equiv \Delta_2^-$ . The last term in the equation for the electric field represents the waveguide losses. The model assumes fixed waveguide losses,  $\ell$ . In addition to the waveguide losses there are also losses upon reflection from the waveguide facet, which are in the case considered here 53% on each end of the laser due to the reflection from the semiconductor air interface, which is not contained in the propagation equations (Eq. (12-15)), but included in the simulation. In fact the low facet reflectivity is the major source of loss in this laser so that other small loss variations in the device due to changes in the operating conditions are expected to be of minor importance and can be safely neglected. In contrast to ref [13], we do not include any effective saturable absorber effects due to potential Kerr-Lensing, since we compare the simulations later with experimental results from devices with a wide ridge (8-20 $\mu\text{m}$ ), where such effects are greatly reduced as discussed in [13].

For simulation purposes, i.e. elimination of noncritical model parameters, we normalize the field with respect to the dipole matrix element, i.e. Rabi frequency, and correspondingly polarization, inversion, inversion grating and pumping as follows:

$$\tilde{E} \equiv \frac{Ed}{\hbar}, \tilde{\eta} \equiv \frac{Nd^2k}{\hbar \epsilon_0 n^2} \eta, \tilde{\Delta}_0 \equiv \frac{Nd^2k}{\hbar \epsilon_0 n^2} \Delta_0, \tilde{\Delta}_2 \equiv \frac{Nd^2k}{\hbar \epsilon_0 n^2} \Delta_2 \text{ and } \tilde{\lambda} \equiv \frac{Nd^2k}{\hbar \epsilon_0 n^2} \lambda.$$

The equations transform to

$$\frac{n}{c} \partial_t \tilde{E}_{\pm} = \mp \partial_z \tilde{E}_{\pm} - i \tilde{\eta}_{\pm} - \ell \tilde{E}_{\pm} \quad (16)$$

$$\partial_t \tilde{\eta}_{\pm} = \frac{i}{2} (\tilde{\Delta}_0 \tilde{E}_{\pm} + \tilde{\Delta}_2^{\mp} \tilde{E}_{\mp}) - \frac{\tilde{\eta}_{\pm}}{T_2} \quad (17)$$

$$\partial_t \tilde{\Delta}_0 = \tilde{\lambda} + i (\tilde{E}_+^* \tilde{\eta}_+ + \tilde{E}_-^* \tilde{\eta}_- - c.c.) - \frac{\tilde{\Delta}_0}{T_1} \quad (18)$$

$$\partial_t \tilde{\Delta}_2^{\pm} = \pm i (\tilde{E}_+^* \tilde{\eta}_- - \tilde{\eta}_+^* \tilde{E}_-) - \left( \frac{1}{T_1} + 4k^2 D \right) \tilde{\Delta}_2^{\pm} \quad (19)$$

For ease of interpretation of the simulation results and matching of model parameters to the experimentally realized operating conditions, which are always related to the threshold pump current, we derive the threshold pumping and threshold inversion for cw-lasing. The continuous wave steady state polarization for  $\tilde{E}_+ = \tilde{E}_- = \tilde{E}$  is

$$\tilde{\eta} = i \frac{T_2}{2} (\tilde{\Delta}_0 + \tilde{\Delta}_2) \tilde{E} \quad (20)$$

which can be substituted in Eq. (16). Thus for the forward propagating field we find

$$\left( \partial_z + \frac{n}{c} \partial_t \right) \tilde{E} = \frac{T_2}{2} (\tilde{\Delta}_0 + \tilde{\Delta}_2) \tilde{E} - \ell \tilde{E}. \quad (21)$$

Thus the gain is  $g = T_2 (\tilde{\Delta}_0 + \tilde{\Delta}_2) / 2$ . At threshold the electric field vanishes and we obtain from Eq. (18) for the inversion  $\tilde{\Delta}_0 = \tilde{\lambda} T_1$  and since there is no grating,  $\tilde{\Delta}_2 = 0$ . We therefore find that the small signal gain is given by  $g_0 = \tilde{\lambda} T_1 T_2 / 2$ . At threshold the gain is equal to the losses, thus the pumping at threshold must be  $\tilde{\lambda}_{th} = 2l / T_1 T_2$  and the inversion at threshold is  $\tilde{\Delta}_{th} = 2l / T_2$ . For the following, we normalize the plots for the inversion always with respect to the threshold inversion  $\tilde{\Delta}_{th}$  for ease of interpretation.

Spatial hole burning is associated with the amplitude  $\tilde{\Delta}_2$  of the inversion grating that couples the electric fields propagating along the two directions in the laser. As can be seen from Eq. (19) the inverse lifetime of the gain grating is given by  $T_g^{-1} = T_1^{-1} + 4k^2 D$ , i.e. it is the sum of the terms due to diffusion of the carrier density modulation and the inverse carrier lifetime, since the latter is the rate by which carriers are homogeneously injected in each volume element. SHB is strong in mid-IR QCLs since the strength of diffusion, which combats the carrier density modulation, scales with the square of the wave number  $k$  [13], which is about an order of magnitude smaller for mid-IR QCLs than for semiconductor lasers in the visible. In steady state, Eq. (18) shows that the term  $i(\tilde{E}_+ \tilde{\eta}_- - \tilde{\eta}_+^* \tilde{E}_-)$  that drives the inversion grating in Eq. (19) scales with  $\tilde{\Delta}_0 / T_1$ . So, in steady state, the strength of the carrier density modulation or gain grating is  $\tilde{\Delta}_2 \sim \tilde{\Delta}_0 / (1 + 4k^2 D T_1)$ . In AlInAs-InGaAs heterostructures, the diffusion coefficient  $D$  is 46 cm<sup>2</sup>/sec at 77 K and for vacuum wavelength 6.2  $\mu\text{m}$ , we obtain  $4k^2 D = 0.2$  THz. In regular mid-IR QCLs ( $T_1 = 5$  ps),  $4k^2 D$  is roughly the same as  $1 / T_1$  and SHB is strong ( $\tilde{\Delta}_2 \approx 0.5 \tilde{\Delta}_0$ ). Even for a longer upper state lifetime, such as  $T_1 = 50$  ps, which is the case for the “superdiagonal” QCL structures discussed here, the effects of SHB cannot be neglected ( $\tilde{\Delta}_2 \approx 0.1 \tilde{\Delta}_0$ ). Due to the fast gain recovery time of QCLs, carrier diffusion cannot suppress SHB, in contrast to standard semiconductor lasers.

## 5. QCL Dynamics

In this section we show and discuss the simulation results, initially without and then with SHB. The parameters that stay fixed and are used in all examples in this paper, if not otherwise explicitly noted, are given in Table 1. It is important to note, that for the QCL with superdiagonal gain structure discussed here, the roundtrip time is almost equal to the gain recovery time,  $T_1$ , a fact that enables stable mode locking as explained before.

**Table 1. Parameters used in simulations if not otherwise noted**

| Quantity                 | Symbol | Value                |
|--------------------------|--------|----------------------|
| Gain recovery time       | $T_1$  | 50 ps                |
| Dephasing time           | $T_2$  | 0.05 ps              |
| Linear cavity loss       | $\ell$ | $10 \text{ cm}^{-1}$ |
| Roundtrip time           | $T_R$  | 56 ps                |
| Roundtrip frequency      | $f_R$  | 17.86 GHz            |
| Cavity length            | $L$    | 2.6 mm               |
| Modulator section length | $L_s$  | 0.24 mm              |
| Facet reflectivity       | $R$    | 53%                  |

### 5.1 QCL Dynamics without SHB

First, the steady state behavior of the laser is investigated when SHB is not included in the Maxwell-Bloch equations (i.e.  $\tilde{\Delta}_2^\pm = 0$ ). Initially the continuous wave steady state solution of the model is determined when the current modulation is switched off. Figure 4 shows the resulting total intracavity intensity and normalized inversion. That means, in each point of the cavity there is a forward and a backward wave and we add the intensities of both waves to the total intensity at each point. The total intensity increases slightly towards the facets of the cavity to compensate for the output coupling losses and as a result the gain is more strongly saturated at the edges than in the center of the cavity.

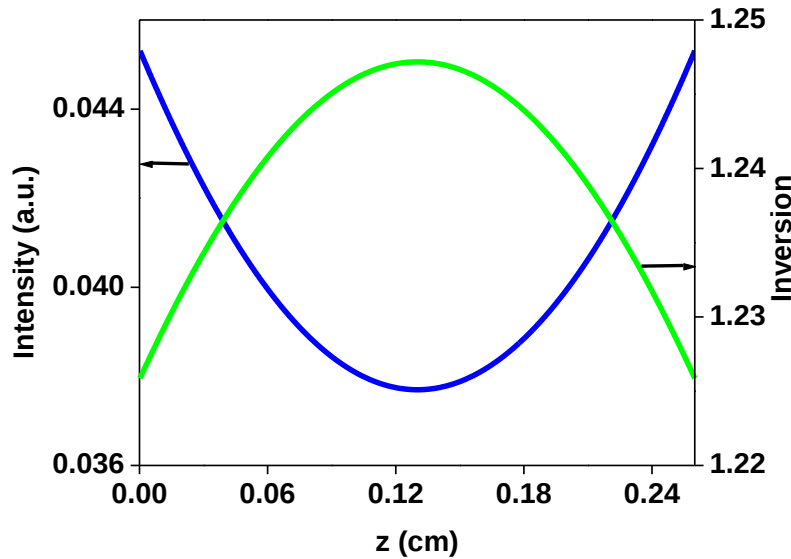


Fig. 4. Steady state intensity (blue line) and inversion (green line) in the cavity without SHB and without modulation for DC pumping  $p = 1.1$  after 1785 roundtrips.

For completeness, Fig. 5 shows the pumping, the inversion and the intensity versus time for a point inside the short (modulated) section. The inversion does not follow the pumping current instantaneously, because the modulation frequency is close to the inverse gain recovery time (i.e.  $f_R = 17.86 \text{ GHz}$  and  $1/T_1 = 20 \text{ GHz}$ ) and as a result the inversion is delayed by almost 90 degrees compared to the pumping. Also, the gain peak does not coincide with the pulse maximum. The front of the pulse extracts gain so it is more amplified than the

back of the pulse experiencing the reduced gain, due to the long gain recovery time compared to the pulse duration. As a result, the group velocity of the pulse is increased.

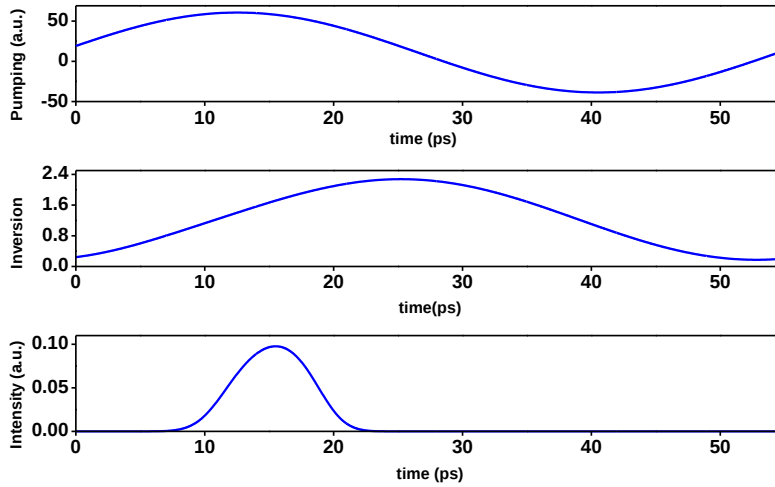


Fig. 5. Pumping, inversion and intensity for a point inside the modulation section for  $p = 1.1$  and  $m = 5$  without SHB.

Second the laser dynamics under current modulation is studied. Figure 6 shows the intensity of the output pulses from the QCL at the right end of the laser cavity, (i.e. the laser output is monitored as a function of time) for different DC pumping levels and constant modulation amplitude ( $m = 5$ ). As expected, isolated pulses with a steady state pulse shape are formed by the strong gain modulation in the short gain section. For increased DC-pumping levels of the long gain section, the pulse energy or average output power is increasing. The pulse duration depends on the pumping level. This can be understood by the fact that the long section acts as an amplifier with a recovery time on the order of the cavity roundtrip time. Although the gain is slow, with a recovery time equal the cavity roundtrip time, it leads to substantial reshaping of the pulse towards longer pulses, i.e. suppression of multimode operation. When the DC pumping level is close to threshold ( $p = 1.1$ ) the pulse is weak and therefore only weakly saturates the amplifier leading to minimum pulse lengthening during propagation through the long section. The long section mostly helps to compensate for the waveguide and facet losses. With increasing DC pumping ( $p = 1.45$  and  $p = 1.61$ ), the pulse energy and with it the gain saturation and pulse lengthening increases, explaining the increase in pulse duration with increased pumping.

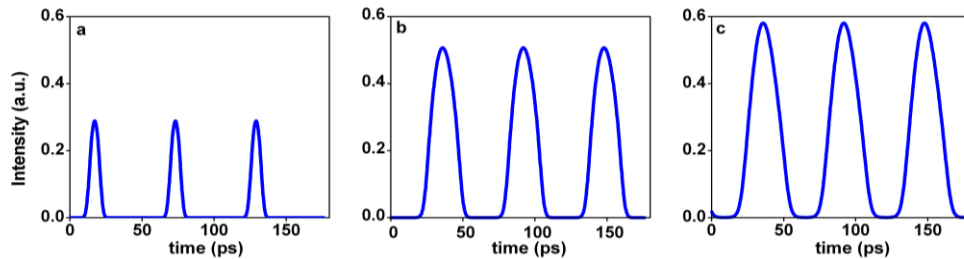


Fig. 6. Intensity profile of output pulse train for modulation with AC-amplitude  $m = 5$  for different DC pumping levels without SHB. a)  $p = 1.1$  b)  $p = 1.45$ , and c)  $p = 1.61$

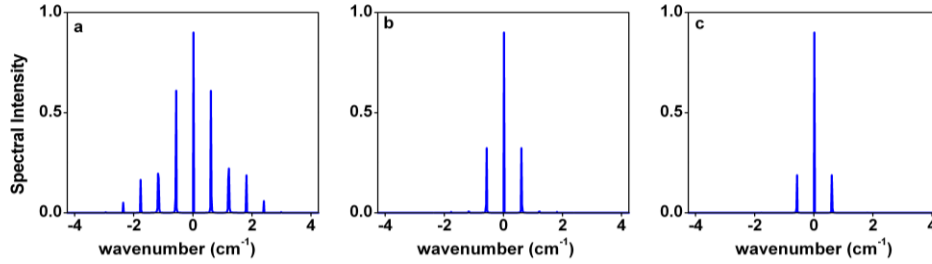


Fig. 7. Spectral intensities for modulation with AC-amplitude  $m = 5$  for different DC pumping levels without SHB a)  $p = 1.1$  b)  $p = 1.45$ , and c)  $p = 1.61$

For the purpose of comparison with later simulation and experimental results, Fig. 7 shows the normalized spectral intensities and Fig. 8 shows the computed Interferometric Autocorrelation (IAC) traces for the three cases in Figs. 6a-6c. When DC pumping is close to threshold many modes are lasing and locked (Fig. 7a) and as a result the pulses are fairly short, as we can see in Fig. 6a. As DC pumping increases we can see in Fig. 6 that the pulses become longer in time and from the spectra we observe that fewer modes are lasing due to the intensity smoothing effect of a saturating and fast recovering gain in the long section. For low DC pumping level the ratio of peak to background is 8:1 (Fig. 8a), which verifies that the pulses are isolated. IAC traces for higher DC pumping levels start to overlap. For the case without SHB, the IAC traces and in fact the computed electric fields do not show any chirp i.e. nontrivial phase over the pulse.

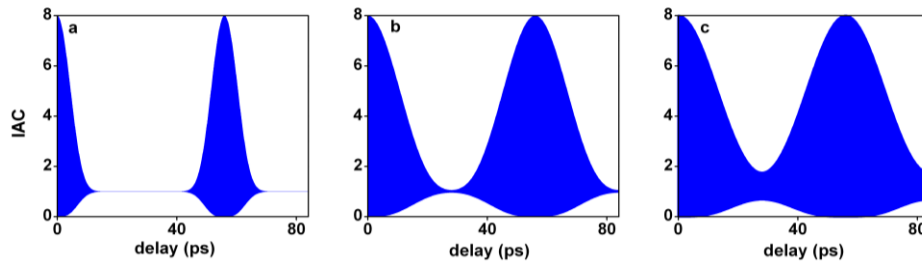


Fig. 8. Interferometric autocorrelation traces (IACs) for modulation with AC amplitude  $m = 5$  for different DC pumping levels without SHB a)  $p = 1.1$  b)  $p = 1.45$ , and c)  $p = 1.61$

As we will see later, these do not fully account for what is found experimentally and inclusion of SHB is necessary to explain the observations.

### 5.2 Experimental observations

The experiments are described in detail in [12]. The experimental results show the same general trend of pulse lengthening with increased DC pumping, however there is a major difference. The pulses in the simulations are in general longer in time than the pulses observed experimentally. For the case of a DC pumping parameter  $p = 1.1$ , the pulse in Fig. 6a is close to a Gaussian with a 7 ps FWHM duration. The measured interferometric autocorrelations for different pumping levels are shown in Fig. 9, and in particular for  $p = 1.1$  in Fig. 9a, assuming a Gaussian pulse shape, a FWHM pulse duration of 3 ps is extracted. The experimentally observed spectra are also much more broadband compared to the theoretical ones from the simulation. This indicates that SHB is important and must be incorporated in the model.

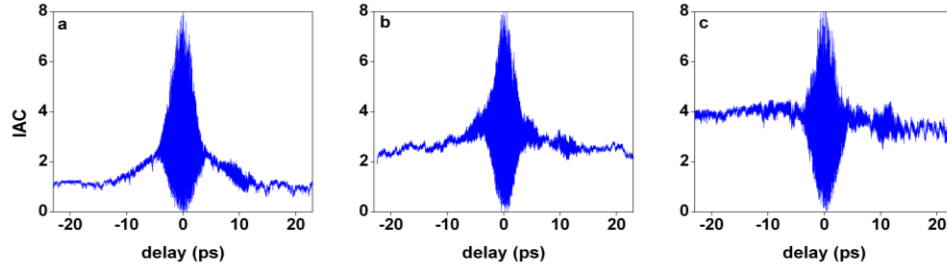


Fig. 9. Measured interferometric autocorrelation traces (IACs) for modulation with AC amplitude  $m = 5$  (35 dBm applied RF-power) for different DC pumping levels. a)  $p = 1.1$  (340 mA) b)  $p = 1.45$  (450 mA), and c)  $p = 1.61$  (500 mA) [12].

### 5.3 QCL Dynamics with SHB

To include SHB in the model we use Eqs. (16-19) in the simulations. The total inversion now consists of the average inversion  $\Delta_0$  and the inversion grating. The interference of the two counter-propagating waves produces a standing wave pattern in the optical intensity, which in turn varies spatially the saturation of the laser medium. Due to the spatial gain grating more modes start lasing, however as we will show below this grating does not become stationary. The wave, as it propagates, is reflected from the nonstationary grating at some positions (SHB interferes with mode locking) and this leads to differential and time varying phase shifts between the modes. Thus, in the presence of SHB the modes acquire nonlinear phase shifts and with it also the pulse. Like in the case without SHB, the DC pumping level should be close to threshold so that the pulses do not lengthen too much.

To obtain insight into the nonstationary gain grating, the continuous wave solution of the model is determined in the absence of a current modulation. In Fig. 10 we show the inversion along the cavity for DC pumping 1.1 times above threshold at three different times about 5000 roundtrips apart. As the number of roundtrips increases, the average inversion and the inversion grating continue to evolve and never reach a steady state. The inversion profile and inversion grating show a degree of randomness. If there would be a steady state, one would expect that due to the large losses per roundtrip the system would certainly come into steady state within at least a hundred roundtrips. This is not the case.

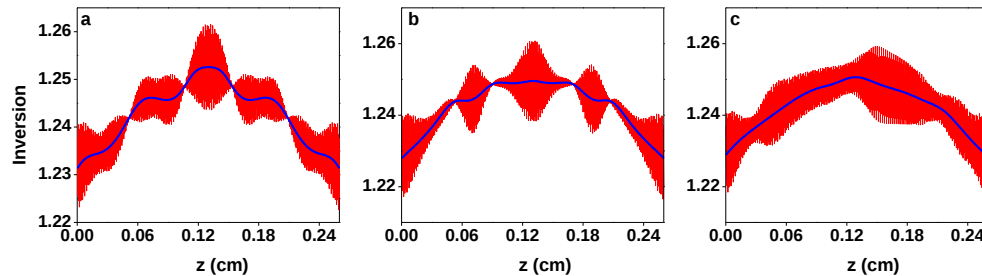


Fig. 10. Average inversion  $\Delta_0$  (blue line) and total inversion along the cavity length including the inversion grating (red line) for DC pumping  $p = 1.1$  and no modulation after a) 5357 roundtrips, b) 10714 roundtrips c) 17857 roundtrips.

If we increase the DC pumping to 2 times above threshold, the degree of randomness in the grating is further increased due to the increase in the number of modes that are lasing, see Fig. 11.

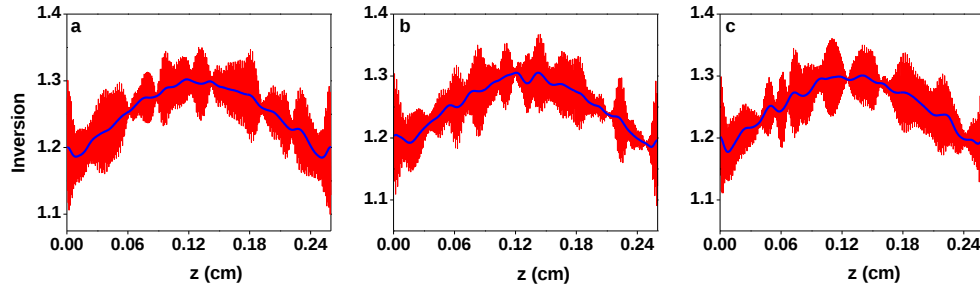


Fig. 11. Average inversion  $\Delta_0$  (blue line) and total inversion along the cavity length including the inversion grating (red line) for DC pumping  $p = 2$  and no modulation after a) 5357 roundtrips, b) 10714 roundtrips c) 17857 roundtrips.

In the following, we show the intensity and the inversion profiles in the cavity when the modulation is switched on, for various time instants and different DC pumping levels. In Fig. 12 the DC pumping is 1.1 times above threshold. When the pulse is reflected from one of the facets, Figs. 12a and 12c, a grating is formed due to the presence of forward and backward waves. Since there are isolated pulses produced, there is no grating when the pulse is propagating in the middle of the cavity, Fig. 12b. However, there is still some gain grating left over at the left end of the waveguide, where the pulse was reflected last and consequently the grating has not yet decayed completely.

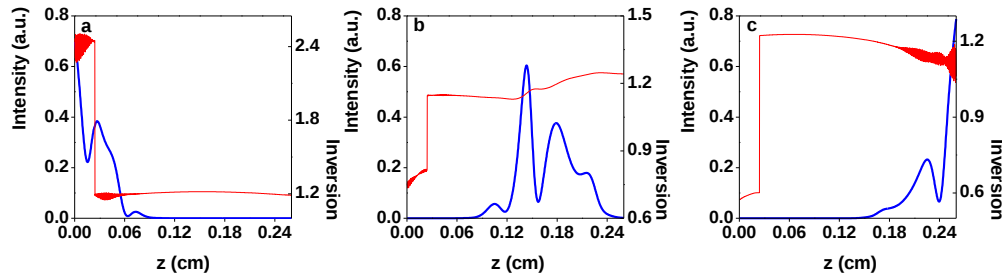


Fig. 12. Intensity profile (blue line) and total inversion (red line) along the cavity for DC pumping  $p = 1.1$  and AC amplitude  $m = 5$  a) when the pulse is reflected off the left facet and propagates to the right b) when the pulse is in the middle of the cavity and propagates to the right c) when the pulse is reflected off the right facet and propagates to the left.

If we increase the DC pumping to 2 times above threshold (Fig. 13) there are no longer isolated pulses per roundtrip. The pulse has broken up into multiple beats of the lasing modes, due to the large and too fast recovering gain in the long waveguide section and a gain grating is observable over the full length of the laser cavity.

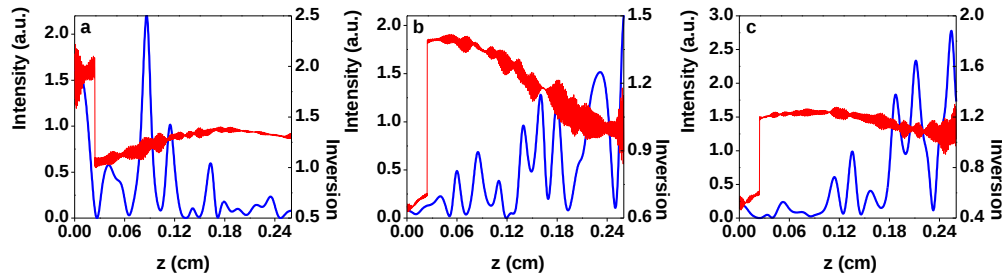


Fig. 13. Intensity profile (blue line) and total inversion (red line) along the cavity for DC pumping  $p = 2$  and AC amplitude  $m = 5$  a) when the pulse is reflected off the left facet and propagates to the right b) when the pulse is in the middle of the cavity and propagates to the right c) when the pulse is reflected off the right facet and propagates to the left.

Note, in both Figs. 12 and 13 there is a discontinuity in the inversion at the boundary between the short modulated section and the long, continuously pumped section.

Similar to Figs. (6-8), we simulate the laser dynamics under current modulation in the presence of SHB. The simulation results shown in the following are snapshots of the laser dynamics after simulating 10,750 roundtrips (0.6  $\mu$ s). Figure 14 shows the normalized time averaged spectral intensities that are generated by Fourier-Transformation of a 29 roundtrip long time series, and Fig. 15 shows the intensity of the output pulses at the end of the QCL cavity for different DC pumping levels and constant modulation amplitude ( $m = 5$ ) for a duration of about three cavity roundtrips. As explained above, the generated pulse trains and spectra do not reach a true steady state, but rather remain dynamic on the few percent level. If we compare the spectral intensities in Fig. 14 with the ones in Fig. 7, where no SHB is included, we see that many more modes are lasing in the presence of SHB in agreement with the experiment. For DC pumping close to threshold ( $p = 1.1$ ) the phases between the modes are locked to the values favoring the formation of an isolated pulse and the main peak of the pulse becomes as short as 2 ps (Fig. 15a). With increasing DC pumping level, more modes start lasing (Figs. 14b, 14c), since the magnitude of the inversion grating becomes stronger. At the same time, the intensity profile of the pulses becomes more structured (Figs. 15b, 15c). The reason for the structure is the decorrelation of the phases between the modes due to SHB. Also the speed of the gain modulation, which stays fixed, is no longer capable to lock all modes of the increasingly broadband spectrum. Furthermore, with increased DC-pumping, passive gain modulation in the amplifier section, due to gain saturation by the pulse and partial gain recovery, becomes stronger and again the peak of the pulse experiences less gain than the wings, destabilizing short pulse generation.

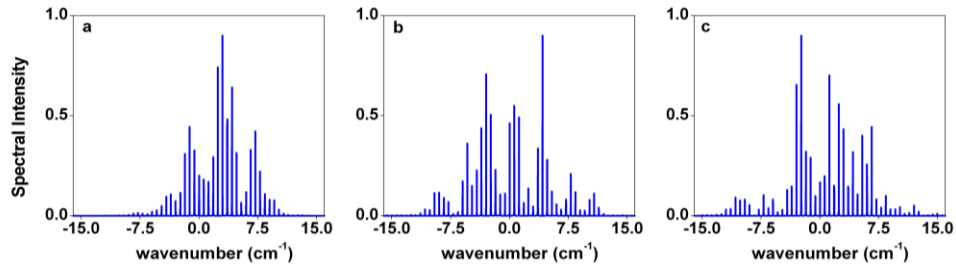


Fig. 14. Spectral intensities for modulation with AC amplitude  $m = 5$  for different DC pumping levels including SHB a)  $p = 1.1$  b)  $p = 1.45$ , and c)  $p = 1.61$

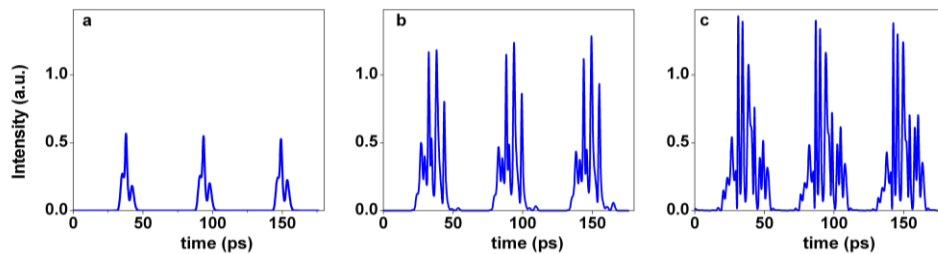


Fig. 15. Intensity profile of output pulse train for modulation with AC amplitude  $m = 5$  for different DC pumping levels including SHB a)  $p = 1.1$  b)  $p = 1.45$ , and c)  $p = 1.61$

The computed IAC traces for the three cases in Fig. 15 are shown in Fig. 16 and reflect the structure that is observed in the intensity profiles. As DC pumping increases, we observe in the IAC traces more and more substructure. The simulation traces back this sub-structure to the nonlinear phase introduced by SHB.

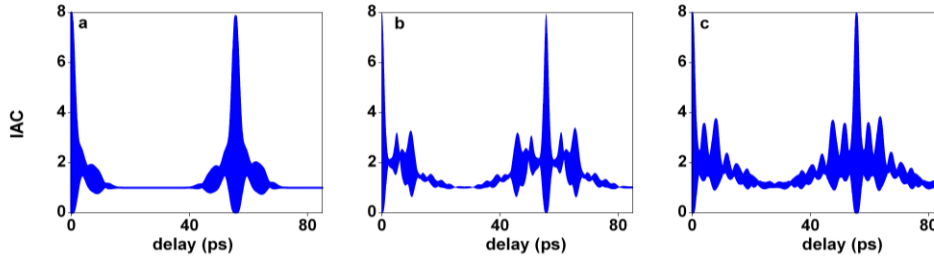


Fig. 16. Interferometric autocorrelation traces (IACs) for modulation with AC amplitude  $m = 5$  for different DC pumping levels including SHB. a)  $p = 1.1$  b)  $p = 1.45$ , and c)  $p = 1.61$

The grating decay time  $T_g$  is always smaller than the upper state lifetime  $T_1$ , see Eq. (19). When the effects of carrier diffusion are negligible, i.e.  $T_g$  becomes close to  $T_1$ , the strength of SHB increases (the amplitude of the inversion grating relaxes slower) and the side lobes in the IAC become more pronounced. In Fig. 17a we see the effect on the side lobes when  $T_g$  increases from 5 ps (used so far) to 10 ps. The same behavior also occurs when  $T_1$  decreases. As explained before, decrease in  $T_1$  results in increase of the effect of SHB and subsequent deterioration of the pulse quality as it is shown in Fig. 17b.

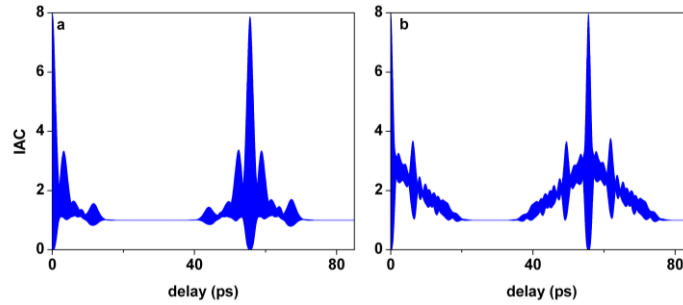


Fig. 17. Interferometric autocorrelation traces (IACs) for modulation with AC amplitude  $m = 5$ , and DC pumping level  $p = 1.1$  a) for  $T_1 = 50$  ps and  $T_g = 10$  ps b) for  $T_1 = 5$  ps and  $T_g = 2.5$  ps. Comparing the two figures with Fig. 13a we see that the structure is more pronounced in these figures due to the stronger SHB.

## 6. Conclusion

We have shown by numerical simulations the limitations in achieving stable short pulse generation from actively mode-locked QCLs, due to the limited upper state lifetime and spatial hole burning. A stable regime of modelocking only occurs for carefully adjusted DC pumping level close to threshold and strong modulation of the pump current in a short modulator section. SHB helps to reduce the pulse duration, but deteriorates pulse quality. Further increase in DC pumping results in pulse lengthening and deterioration of pulse quality both due to spatial hole burning and increased gain modulation due to gain saturation by the pulse and subsequent partial gain recovery. Simulations and experimental results show the same trend as DC pumping increases.

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